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DYNAMIC MODELING AND STABILITY ANALYSIS OF A LIQUID ROCKET ENGINE

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Abstract. *The stability of a liquid rocket engine (LRE) has been extensively studied in foreign space programs, mainly because the combustion chamber, by itself, is a source of unstable processes. The next phase of the Brazilian space program requires a reliable engine to fulfill the mission goals. Thus, understanding and predicting the influence of self-oscillating process is a necessity. In the present work, it is modeled several components of a LRE, which is able to be part of the upper stage of the Brazilian VLS-2 rocket. The combustion chamber, the injector head, the cooling jacket and the pipelines constitute a system that is simulated by means of the MATLAB/SIMULINK. Afterwards, the stability of the system is analyzed for three parameters of great importance for such analysis. The stated problem is solved by the construction of a region of stability, using two criteria, Mikhailov and Hermite-Biehler, to find out the stability limits.*

Keywords: *Liquid rocket engine, liquid propulsion, rocket engine, dynamic modeling, stability analysis.*

1. INTRODUCTION

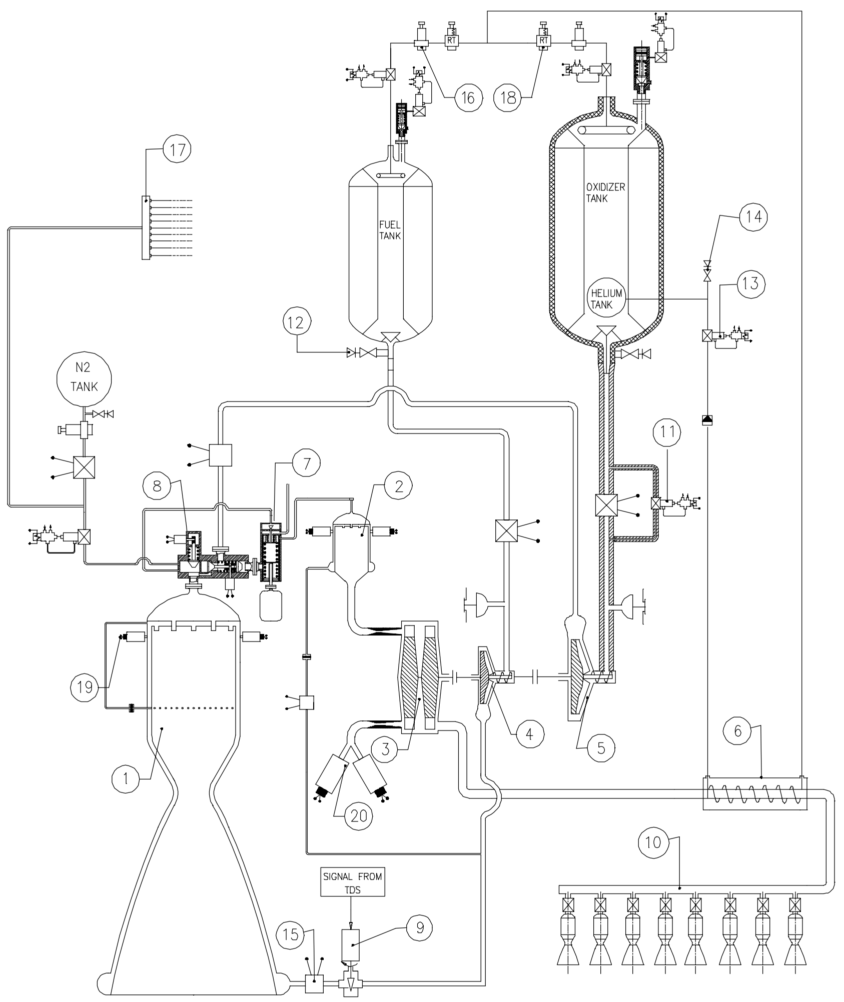
A Liquid Rocket Engine (LRE) contains several sources of intense pressure fluctuations caused by turbulent flow in feed line, fluttering of pump wheel blades, vibrations of control valves, unsteady motion in combustion chamber and the gas generator (Volkov *et al*., 1978). The result of the coupling of these oscillations with the natural frequencies of the system is the generation of instability, which is often observed with catastrophic consequences.

The main aim of the present work is the development of mathematical models for several units of a typical LRE system, namely, the combustion chamber, the injector head, the cooling jacket, the mixture ratio regulator and the propellant pipelines. The modeling of the system is made by using the knowledge of its working principles and the characteristics of the LRE hydropneumatic system. The simulation, step response, is carried out by means of the software MATLAB/ SIMULINK. The system is simulated by taking into account two different models, with and without the effect of fluid compressibility and pipeline elasticity.

In addition to its dynamical characteristics, the system stability is analyzed for three selected parameters, inside a range of values, so that the system is stabilized. The Mikhailov and the Hermite-Biehler criteria are used to find these limits of stability.

2. HYDROPNEUMATIC SCHEME OF THE ROCKET ENGINE

The hydropneumatic system of a LRE has a great number of components, including: a combustion chamber, turbopump unit, gas generator, tanks, valves, regulators, pipelines, and thrust vector control nozzles (see Fig. 1).



1- Combustion chamber; 2- Gas generator; 3- Turbine; 4- Fuel pump; 5- Oxidizer pump; 6- Heat exchanger; 7- Thrust regulator; 8- Main oxidizer valve; 9- Mixture ratio regulator; 10- Thrust vector control; 11- Electro-pneumatic valve for the pre-cooling system; 12- Propellant filling valve; 13- Electro-pneumatic valve for the helium feeding; 14- Helium filling valve; 15- Fuel cut-off valve; 16- Pressure regulator; 17- Nitrogen distributor for pressurizing the electro-pneumatic valve; 18- Temperature regulator; 19- Pyrotechnical igniter; 20- Pyrotechnical cartridge.

Figure 1- Hydropneumatic system of the rocket engine

3. THE SYSTEM MODELING

The modeling of the system, at the nominal regime of operation, is executed by using the knowledge of the working principles and the main characteristics of the LRE hydropneumatic system, Fig.1. The main component blocks are *combustion chamber, injector head, mixture ratio regulator, feed line and the cooling jacket.*

3.1 Combustion chamber

The combustion chamber is a device where oxidizers and fuels are mixed and burned to produce hot gases that are expanded through a convergent-divergence nozzle, producing the desired thrust. The combustion gases are assumed ideal, and the steady flow in the nozzle isentropic and the propellants are ideally stirred. The pressure in the combustion chamber (ΔP_{cc}) is the output parameter, the fuel mass flow rate ($\Delta m_{f.cc}$) and oxidizer ($\Delta m_{o.cc}$) are the input parameters of the nominal regime. The Laplace transform of the combustion chamber equation, while considering the time delay effect, (τ_{cc}), is obtained using the mass equilibrium equation of the nominal regime of operation and the *energy coefficient* (Φ_{cc}) to represent the burning and nozzle efficiency (Kessaev, 1997)

$$(T_{ch} s + 1) \cdot \overline{\Delta P_{cc}} = e^{-s\tau_{cc}} \cdot \left(K_1 \cdot \overline{\Delta m_{o.cc}} + K_2 \cdot \overline{\Delta m_{f.cc}}\right). \tag{1}$$

According to Gladkova (1997), the burning process time constant, (T_{ch}), the gain constants for the oxidizer line (K_1) and for the fuel line (K_2) are given by:

$$T_{ch} = \frac{V_{cc} \cdot C_{star.cc}}{R_{cc} \cdot T_{cc} \cdot A_{cr}} \cdot \phi_{cc}. \tag{2}$$

$$K_1 = \frac{K_{m.cc}}{K_{m.cc} + 1} \tag{3}$$

$$K_2 = \frac{1}{K_{m.cc} + 1}. \tag{4}$$

where A_{cr} is the throat area, $C_{star.cc}$ is the characteristic velocity of gas, V_{cc} is the CC volume, T_{cc} is the gas temperature, R_{cc} is the gas constant and $K_{m.cc}$ is the propellant mixture ratio (O/F).

3.2 Injector head

The fuel injection is an essential process of the LRE because all feedback couplings of the combustion chamber are realized (Bazarov & Yang, 1998). Here, the injector head is considered a constriction of fixed area ($A_{inj.i}$), from basic fluid mechanics it is possible to derive the orifice flow equations using energy, momentum, and mass continuity laws (Kessaev, 1997). The main result is that the injector pressure drop varies with the square of the flow rate of liquid propellants.

The mass flow rate through each injector ($\Delta m_{inj.i}$) is the output parameter, the pressure of propellant injection ($\Delta P_{inj.i}$) and combustion chamber (ΔP_{cc}) are the input parameters. It is assumed that the propellant is an incompressible liquid, the walls of the injector are rigid and the heat transfer problem is neglected. The Laplace transform, for the *i*-propellant injectors is:

$$\overline{\Delta m_{inj.i}} = K_3 \cdot \overline{\Delta P_{inj.i}} + K_4 \cdot \overline{\Delta P_{cc}}. \tag{5}$$

The gain constants K_3 and K_4 for *i*-propellant injector are:

$$K_3 = \left(\frac{1}{2} N_{inj} \cdot \mu_i \cdot A_{inj.i} \sqrt{\frac{2\rho_i}{(P_{inj.i} - P_{cc})}}\right) \cdot \frac{P_{inj.i}}{m_{inj.i}}. \tag{6}$$

$$K_4 = \left(-\frac{1}{2} N_{inj} \cdot \mu_i \cdot A_{inj.i} \sqrt{\frac{2\rho_i}{(P_{inj.i} - P_{cc})}} \right) \cdot \frac{P_{cc}}{m_{i.cc}} . \quad (7)$$

where ρ_i is the propellant density, $N_{inj.i}$ is the number of injectors, $\mu_{inj.i}$ is the discharge coefficient of the injector and the subscripts, i = o, f, stand for oxidizer and fuel respectively.

3.3 Throttle of mixture ratio regulator

For simplicity, the modeling of the throttle mixture ratio regulator follows the same approach as the injector head model. The liquid is considered incompressible, with rigid walls. Besides, the cavitation and the mass flow blocking due to high flow acceleration in the throttling device are not considered here.

The throttle acts as an orifice with a variable area $A(x)$, where x represents the valve opening coordinate, controlled by an electrical driver. The input parameters for the regulator are the inlet pressure ($P_{in.reg}$), the mass flow rate through ($m_{f.cc}$) and the regulating area (μF_{reg}). The output parameter is the outlet pressure ($P_{out.reg}$). Thus, the Laplace transform is written as:

$$\overline{\Delta P_{out.reg}} = K_{10} \cdot \overline{\Delta P_{in.reg}} + K_{11} \cdot \overline{\Delta m_{f.cc}} + K_9 \cdot \overline{\Delta \mu F_{reg}} . \quad (8)$$

where the gain constants for throttle of mass ratio (K_9, K_{10} and K_{11}) are:

$$K_9 = \left(\frac{m_{f.cc}^{\ 2}}{\rho_f \cdot (\mu F_{reg})^3} \right) \cdot \frac{\mu F_{reg.N}}{P_{out.reg.N}} . \quad (9)$$

$$K_{10} = \left(\frac{P_{in.reg}}{P_{out.reg}} \right) . \quad (10)$$

$$K_{11} = \left(\frac{-m_{f.cc}}{\rho_f \cdot (\mu F_{reg})^2} \right) \frac{m_{f.cc}}{P_{out.reg}} . \quad (11)$$

and where ρ_f is the fuel density, μF_{reg} is the product of the discharge coefficient times the area of regulator passage.

3.4 Cooling jacket and pipelines

The flow in the pipelines is considered as one-dimensional and the influence of the acceleration in the flow is taken into account, as an inertia term. The friction is considered as a fluid resistance. The discharge coefficient ($\mu_{pipe.i}$) is obtained experimentally, it can vary from 0.6 to 0.9 (Kessaev, 1997). The liquid density ($\rho_{pipe.i}$) is considered constant or variable, depending on the model. The Laplace transform for i- propellant pipeline considering the liquid compressibility and the pipe elasticity is written as:

$$(I_{pipe.i} C_{pipe.i} s^2 + R_{pipe.i} C_{pipe.i} s + 1) \cdot \overline{\Delta P_{inj.i}} = K_{15} \cdot \overline{\Delta P_i} + K_{16} \cdot (T_{pipe.i} s + 1) \cdot \overline{\Delta m_{pipe.i}} . \quad (12)$$

where the gain constant K_{15} and K_{16}, and the time constant of i-propellant pipeline ($T_{pipe.i}$) are, respectively:

$$K_{15} = \left(\frac{P_{i.N}}{P_{inj.i.N}} \right). \tag{13}$$

$$K_{16} = -\frac{(m_{pipe.i.N})^2}{P_{inj.i.N} \cdot \rho_i \cdot (\mu_{pipe.i} A_{pipe.i})^2} = -\frac{2 \cdot (P_i - P_{inj.i})}{P_{inj.i}}. \tag{14}$$

$$T_{pipe.i} = -\frac{L_{pipe.i} m_{pipe.i}}{A_{pipe.i} P_{inj.i}} \cdot \frac{1}{K_{16}} \tag{15}$$

For a given tube with equivalent length ($L_{pipe.i}$): $P_{inj.i}$ (P_i) represents the pressure at the end (beginning) of the pipe; $m_{pipe.i}$ (m_i) is the mass flow rate at the end (beginning); $A_{pipe.i}$ is the cross-sectional area; $V_{pipe.i.}$ is the pipe (liquid) volume; ($I_{pipe.i}$) is the fluid inertia in the pipe segment, (C_{eq}) is the equivalent capacitance, ($R_{pipe.i}$) is the liquid resistance. They are written as below:

$$I_{pipe.i} = \frac{L_{pipe.i} \rho_i}{A_{pipe.i}}. \tag{16}$$

$$C_{eq.i} = \frac{V_{pipe.i}}{\beta_{LOx}} + V_{pipe.i} \cdot \frac{2 r_{pipe.i}}{E t_{w.i}}. \tag{17}$$

$$R_{pipe.i} = \frac{m_{i.cc}}{(\mu_{pipe.i} A_{pipe.i})^2}. \tag{18}$$

where E is the elastic modulus of the tube; $t_{w.i}$ is the thickness of the tube wall, $r_{pipe.i}$ is the internal radius of pipe and β_{Lox} is the *Bulk modulus* for Lox or kerosene.

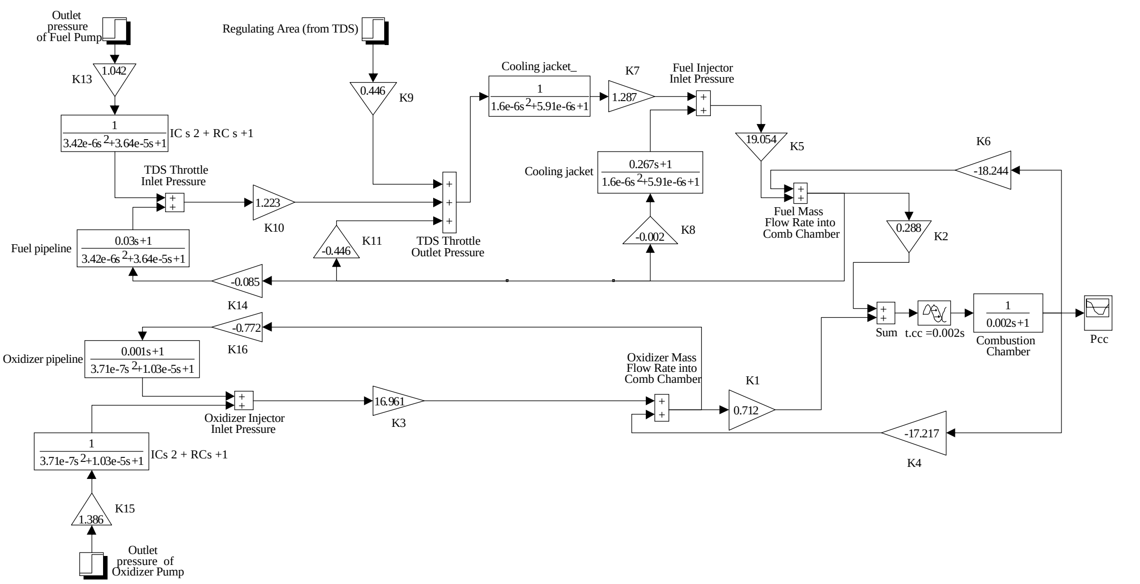


Figure 2 – Block diagram of the model considering the liquid compressibility

The mathematical models are used to simulate many aspects of engine system operation.

It requires a numerical definition of all parameters of the model, time constants and the gains. The block diagram representation (Fig. 2) takes into account the liquid compressibility and pipeline elasticity. A simplified model is obtained when the effects of the pipelines and cooling jacket are neglected.

4. ANALYSIS OF DYNAMIC CHARACTERISTICS OF THE SYSTEM

The performance of dynamic systems in the time domain can be defined in terms of the time response to a step function input, i.e., the step response.

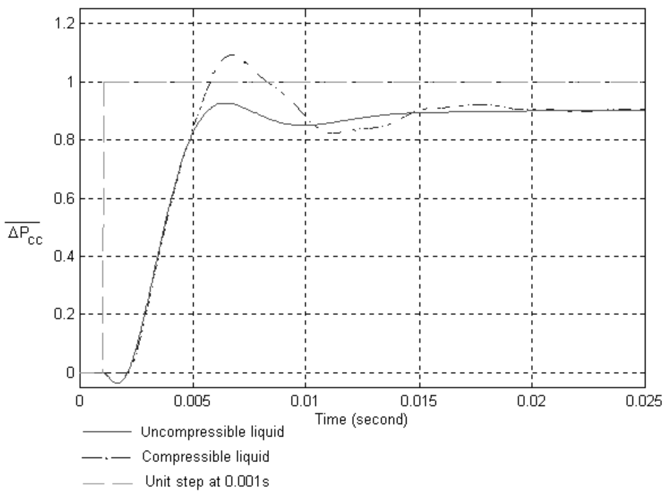


Figure 3- Comparison of the step response for the compressibility effect

Figure 3, shows the step response comparing the effects of liquid compressibility and pipeline elasticity in the pipelines and the cooling jacket. This comparison shows these effects can be neglected, at least when the LRE works with liquid oxygen and kerosene, because these propellants have large moduli of compressibility.

5. STABILITY ANALYSIS OF THE SYSTEM

The most interesting task, in the study of stability process, is the analysis of the influence of system's parameters in its stability. The stated problem is solved by the construction of a region of stability, i.e. determining which combination of parameters defines the limit of stability. Here, two methods are applied to construct this region: Mikhailov and Hermite-Biehler.

The three selected parameters have the greatest influence on the development of low frequency oscillations, i.e., a small change in these parameters are able to take the system to instability. The pressure drop in the injector head (ΔP_{inj}) establishes the connection between chamber and flow oscillations. The time delay of combustion chamber (τ_{cc}) is greatly influenced by the propellant properties. Decreasing it, permits the increase of stability. The time constant of combustion chamber (T_{ch}) is numerically defined as the time that the gas stays inside the chamber, and also has influence on stability (Volkov *et al.*, 1978).

5.1 Stability region by Mikhailov Criterion.

The Mikhailov criterion is a classical tool for stability analysis of a nominal plant (Gladkova, 1997). It requires construction of a frequency plot of the characteristic polynomial. For this criterion, the system is located at the stable region if the frequency plot of the characteristic equation, $M(\omega)=P(\omega)+jQ(\omega)$, with ω from zero to infinity, encircles the origin of the coordinate system (0,0) in a counterclockwise direction.

The linear dynamic behavior of combustion chamber at the nominal working regime, for a constant propellant mixture ratio is described by:

$$T_{ch} \cdot \frac{d(\delta P_{cc})}{dt} + \delta P_{cc} - \delta m_{cc} \cdot (t - \tau_{cc}). \tag{19}$$

The total mass flow rate into combustion chamber (m_{cc}) is a function of pressure difference between combustion chamber (P_{cc}) and injector head (P_{inj}):

$$m_{cc} = a \cdot \sqrt{P_{inj} - P_{cc}} \tag{20}$$

The coefficient a is defined by the hydraulic characteristics of injector head and propellant density. From Eq. (19) and after linearization of the Eq. (20), it is obtained:

$$\delta m_{cc}(t - \tau_{cc}) = \frac{-\delta P_{cc}(t - \tau_{cc})}{\overline{\Delta P_{inj}}} \tag{21}$$

$$\overline{\Delta P_{inj}} = \frac{2 \cdot (\overline{P_{inj}} - \overline{P_{cc}})}{\overline{P_{cc}}} \tag{22}$$

The characteristic equation is obtained the by the Laplace transform of Eq. (21):

$$M(s) = T_{ch} \cdot s + \frac{1}{\overline{\Delta P_{inj}}} \cdot e^{-s \cdot \tau_{cc}} + 1 = 0 \tag{23}$$

The parameters of stability are T_{ch}, ΔP_{inj} and τ_{cc}, whose values determine the roots of the characteristic equation, i.e., the stability limits. Substituting the operator $s=j\omega$, in Eq. (23), and changing the function representation to Euler's form, it is possible to obtain the equation for stability limits:

$$\overline{\Delta P_{inj}} + cos(\omega\tau_{cc}) + j\left(T_{ch} \cdot \overline{\Delta P_{inj}}\omega - sin(\omega\tau_{cc})\right) = 0 \tag{24}$$

By means of Eq. (24), the limit of stability can be build as function of two parameters: ΔP_{inj} -τ_{cc}, ΔP_{inj} -T_{ch} or T_{ch}-τ_{cc}. These parameters of interest are isolated to simplify the calculation, and the result is:

$$\overline{\Delta P_{inj}} = \frac{1}{\sqrt{1 + (T_{ch}\omega)^2}} \tag{25}$$

$$\tau_{cc} = \frac{1}{\omega}\left(k\pi - arctg(\omega T_{ch})\right) \tag{26}$$

each value of k (0,1,2,3,4,...) corresponds to a limit of stability. For simplification, it is considered the case when k=1. The result of Mikhailov criterion to find the limits of stability is shown in Figure 4. The parameters of interest are isolated (ΔP_{inj} -τ_{cc}) and the third parameter (T_{ch}) is used to establish the different curves. All the points under the limit of stability, for example the points in the axis of relative pressure drop of injector head, are considered inside of the stable region.

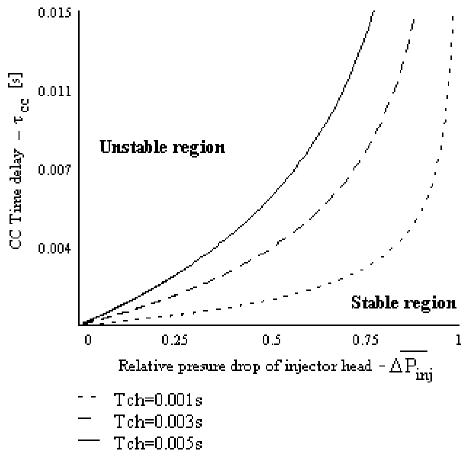


Figure 4- Boundaries of stability of combustion chamber for different values of T_{ch}.

Varying ω from zero to infinity it is possible to build the limit of stability, which divide the plane into two parts; this is showed in the Fig. 4. In one part the set of values of ΔP_{inj} and τ_{cc} assure that the process is stable, and in another one is not stable.

The Mikhailov criterion sometimes can offer some obstacles for obtaining a region of stability. Trying to overcome these barriers, it is presented an analytical solution for the problem of stabilizing a given system. The method is based on the Hermite-Biehler theorem that provides necessary and sufficient conditions for the system stability.

5.2 The stability limits by the Hermite-Biehler Theorem.

The Hermite-Biehler theorem, also called Interlacing theorem provides necessary and sufficient conditions for the Hurwitz stability of a given real polynomial (Ho *et al.*, 1997).

Here, it is reproduced without proof the algorithm to determine the limits of stability for a liquid rocket engine. The characteristic equation of the model considering the liquid incompressible is analyzed for two unknown parameters, T_{ch} and τ_{cc}.

Limits for the time delay of combustion chamber (τ_{cc}). For the mathematical model of the liquid rocket engine, taking into account the time delay and liquid inertia, the characteristic equation is given by:

$$\begin{aligned}\delta(s,\tau_{cc}) = 914.013 + 5.84538s + 0.07813366s^2 + 4.47436\times10^{-6}s^3 \cdots \\ + (-47.7664s + 0.684032s^2 + 0.00390683s^3 + 2.23718\times10^{-6}s^4)\tau_{cc}\end{aligned} . \quad (27)$$

The time delay of combustion chamber (τ_{cc}) is the parameter of analysis to find the range of stability. For convenient nomenclature it is assumed:

$$\delta(s,\tau_{cc}) = D(s) + \tau_{cc}\cdot N(s).$$

Both polynomials of the characteristic equation contain even and odd terms:

$N(s) = N_e(s) + sN_o(s)$ and $D(s) = D_e(s) + sD_o(s)$.

For applying the Interlacing Theorem, it is necessary to exclude all the odd terms of *N(s)*, i.e., *N(s)* must be a real number. The easiest way is making $\delta(s,\tau_{cc})$ times *N'(s)*, where *N'(s)=N(-s)*. It follows that:

$$\delta(s,\tau_{cc})\cdot N'(s) = 43659.110s - 2281.629\tau_{cc}s^2 + 904.427s^2 + 4.159s^3 + 0.8411\tau_{cc}s^4 \cdots$$
$$+ 3.287\times 10^{-2}s^4 - 2.891\times 10^{-4}s^5 - 1.220\times 10^{-5}\tau_{cc}s^6 + 1.573\times 10^{-7}s^6 \cdots$$
$$+ 1.001\times 10^{-11}s^7 + 5.005\times 10^{-12}\tau_{cc}s^8 \quad (28)$$

Afterwards, the real and imaginary parts of the Eq. (28) are separated and it is made $s = j\omega$:

$$\delta(j\omega,\tau_{cc})\cdot N'(j\omega) = p_1(\omega) + \tau_{cc}\cdot p_2(\omega) + j\cdot q(\omega) \quad (29)$$

The main idea is to study the root distribution of the Eq. (28) to know how many roots are located in the right half of the complex plane. By means of the Lemma 5.1 (Ho *et al.*, 1997), δ(s) is Hurwitz if and only if:

$$\sigma(\delta(s,\tau_{cc})\cdot N'(s)) = n - \sigma(N'(s)) \quad (30)$$

Where the "signature" of *N'(s)*, $\sigma(N'(s))$, is defined as: $\sigma(N'(s)) = l' - r'$. From Definition 3.1 (Ho *et al.*, 1997) *l'* and *r'* are the roots number of *N'(s)* on the left-hand side and right-hand side of complex plane, respectively. There are two real, non-negative distinct finite zeros of $q(\omega)$, hence l = 2. Thus: $\sigma(N'(s)) = l$.

From Definition 5.1 (Ho *et al.*, 1997), Since $n + m'$ = 8 is even and l =2, from the stated definition, the set *A* for all possible strings $\{i_0, i_1, i_2\}$ is:

$$A = \begin{Bmatrix} \{-1,-1,-1\} & \{0,-1,-1\} & \{1,-1,-1\} \\ \{-1,-1,1\} & \{0,-1,1\} & \{1,-1,1\} \\ \{-1,1,-1\} & \{0,1,-1\} & \{1,1,-1\} \\ \{-1,1,1\} & \{0,1,1\} & \{1,1,1\} \end{Bmatrix}$$

Here $\sigma(\delta(s,\tau_{cc})\cdot N'(s)) = 3$.

According to the Definition 5.3 (Ho *et al.*, 1997) and $(-1)^{2-1}\cdot sgn(q(\infty)) = 1$, it is necessary to find the subset of strings *F** satisfying: $i_0 - 2i_1 + i_2 = 3$. Hence F* = $\{I_1\}$ where I_1={0, -1, 1}.

The next step is finding the range of τ_{cc} which leads the system into the region of stability. Each string must satisfy the following condition: $(p_1(\omega_t) + \tau_{cc}p_2(\omega_t))\cdot i_t > 0$, $t = 0\ldots 2$. It is necessary define the third frequency as infinity ($\omega_2 = \infty$). For I_1 = {0, -1, 1}:

$$\tau_{cc} < -\frac{p_1(\omega_1)}{p_2(\omega_1)} = 0.036 \qquad \text{and} \qquad \tau_{cc} > -\frac{p_1(\omega_2)}{p_2(\omega_2)} > 0$$

For $\omega_0 = 0$, $p_1(\omega_0) = p_2(\omega_0) = 0$, this case does not impose any additional constraint on τ_{cc}. Thus, it follows from Theorem 4.1 (Ho *et al.*, 1997) that the stabilizing τ_{cc} values must satisfy the string of inequalities above:

$$0 < \tau_{cc} < 0.036.$$

It is possible to check the roots of characteristic equation for some values of τ_{cc}. When the real part of a root is positive, so it is ensured that the system is unstable. Otherwise, it is stable.

Applying the same procedure used to find the limits for the time delay in the characteristic equation as function of time constant of combustion chamber (T_{ch}):

$$\begin{aligned}\delta(s,T_{ch}) &= s^3 + 2245.66s^2 + 4.45\times10^6 s + 8.17\times10^8 \cdots \\ &\quad + \left(s^4 + 3246.32s^3 + 2.67\times10^6 s^2 + 3.65\times10^8 s\right)\cdot T_{ch}\end{aligned} \quad (31)$$

It follows from (Ho *et al.*, 1997) that the stabilizing T_{ch} values must satisfy the following inequality: $T_{ch} > 0$.

It is possible to check the roots of characteristic equation for some values of T_{ch}. When the real part of the root is positive, the system is unstable. Otherwise, it is stable. The value for the time delay of combustion chamber is fixed (τ_{cc} =0.002s) and it is used the first order of Padé approximation.

Undoubtedly, the greatest advantage of the Hermite-Biehler Theorem is the possibility of finding the limits of stability for any numbers of unknown parameters, since they appear linearly in the coefficients of characteristic equation. Unfortunately, the polynomial, regarding the two analyzed parameters, has non-linear coefficients.

6. CONCLUSION

The dynamic analysis of the LRE system model composed of a combustion chamber, injector head, cooling jacket, thrust regulator and fuel pipelines lead to following conclusions. The modeling of the system is appropriate since the main factors have been considered, it was suspected that the liquid compressibility would be another important factor of influence on the system dynamic. The comparison between the models with and without the compressibility effect, by means the step response curves, showed that this influence could be neglected.

The region of stability was obtained, for the three parameters (τ_{cc}, T_{ch} and ΔP_{inj}) by means of two different methods, Mikhailov and Hermite-Biehler criteria. Each method has its own peculiarities and applicability. However, the Hermite-Biehler criterion is the most simple to implement, because it is not necessary to work with complex number (Mikhailov). The Hermite-Biehler provided the limits of stability for the time delay and the system time constant, and these limits were greater than those obtained by the Mikhailov.

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